



ECE 111
Electronic Engineering Fundamentals
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Equilibrium Carrier Concentrations

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Maxwell Boltzmann (M-B) approximation



- Simplified M-B expression for the number of electrons in the conduction band:

$$n = N_C \exp\left(\frac{E_F - E_C}{KT}\right) \quad N_C = \frac{4\sqrt{2}}{h^3} (KT \pi m_e^*)^{3/2}$$

- N_C = Effective density of states in the conduction band. It varies as $T^{3/2}$ and $(m_n/m_o)^{3/2}$

$$N_C = 2.5 \times 10^{19} \left(\frac{m_e^*}{m_o}\right)^{3/2} \left(\frac{T}{300}\right)^{3/2} \text{ cm}^{-3}$$

- E_F = Fermi energy. We do not need its **absolute** value. We only need its **relative position** with respect to the conduction band minimum, E_C . If we know $(E_C - E_F)$, we can calculate n and vice-versa.



- Similarly, the number of holes in the valence band is given by:

$$P = N_V \exp\left(\frac{E_V - E_F}{KT}\right) \quad N_V = \frac{4\sqrt{2}}{h^3} (KT \pi m_h^*)^{3/2}$$

- N_C – Effective density of states in the valence band. It varies as $T^{3/2}$ and $(m_h^*/m_o)^{3/2}$

$$N_V = 2.5 \times 10^{19} \left(\frac{m_h^*}{m_o}\right)^{3/2} \left(\frac{T}{300}\right)^{3/2} \text{ cm}^{-3}$$

Constants



Physical constants

a_B	=	0.5292 Å	Bohr radius	($a_B = 0.5292 \times 10^{-10} \text{ m}$)
ϵ_0	=	$8.8542 \times 10^{-12} \text{ A s / (V m)}$	dielectric permittivity in vacuum	
e	=	$1.6022 \times 10^{-19} \text{ C}$	elementary charge	
c	=	$2.9979 \times 10^8 \text{ m / s}$	velocity of light in vacuum	
E_{Ryd}	=	13.606 eV	Rydberg energy	
g	=	9.8067 m / s ²	Acceleration on earth at sea level due to gravity	
G	=	$6.6873 \times 10^{-11} \text{ m}^3 / (\text{kg s}^2)$	Gravitational constant ($F = G M m / r^2$)	
h	=	$6.6261 \times 10^{-34} \text{ J s}$	Planck constant	($h = 4.1356 \times 10^{-15} \text{ eV s}$)
\hbar	=	$1.0546 \times 10^{-34} \text{ J s}$	$\hbar = h / (2\pi)$	($\hbar = 6.5821 \times 10^{-16} \text{ eV s}$)
k	=	$1.3807 \times 10^{-23} \text{ J / K}$	Boltzmann constant	($k = 8.6175 \times 10^{-5} \text{ eV / K}$)
μ_0	=	$1.2566 \times 10^{-6} \text{ V s / (A m)}$	magnetic permeability in vacuum	
m_e	=	$9.1094 \times 10^{-31} \text{ kg}$	free electron mass	
N_{Avo}	=	$6.0221 \times 10^{23} \text{ mol}^{-1}$	Avogadro number	
$R = k N_{\text{Avo}}$	=	$8.3145 \text{ J K}^{-1} \text{ mol}^{-1}$	ideal gas constant	

Useful conversions

$$1 \text{ eV} = 1.6022 \times 10^{-19} \text{ C V} = 1.6022 \times 10^{-19} \text{ J}$$

$$kT = 25.86 \text{ meV (at } T = 300 \text{ K)}$$

$$E = h\nu = hc/\lambda = 1239.8 \text{ eV nm} / \lambda$$

Worked Example:



The effective mass of electrons in the conduction band of Si is $1.18m_0$ where m_0 is the free electron mass. Calculate the effective density of states in the conduction band of Si at 300 K and 500 K. If the Fermi energy in a sample of Si is 0.1 eV and 0.18 eV below the conduction band edge at 300 K and 500 K, respectively, calculate the electron concentrations in this sample at these two temperatures. Boltzmann constant $k = 8.617 \times 10^{-5}$ eV/K.

Solution:

$$N_C (300 \text{ K}) = 2.5 \times 10^{19} \times 1.18^{3/2} \times 1^{3/2} = 3.2 \times 10^{19} \text{ cm}^{-3}.$$

$$n (@300 \text{ K}) = 3.2 \times 10^{19} \times \exp\{-0.1 / (8.617 \times 10^{-5} \times 300)\} = 6.7 \times 10^{17} \text{ cm}^{-3}.$$

$$N_C (500 \text{ K}) = 2.5 \times 10^{19} \times 1.18^{3/2} \times (500/300)^{3/2} = 6.9 \times 10^{19} \text{ cm}^{-3}.$$

$$n (@ 500 \text{ K}) = 6.9 \times 10^{19} \times \exp\{-0.18 / (8.617 \times 10^{-5} \times 500)\} = 1.06 \times 10^{18} \text{ cm}^{-3}.$$

Worked Example



The effective mass of holes in the valence band of Si is $0.81m_0$ where m_0 is the free electron mass. Calculate the effective density of states in the valence band of Si at 300 K and 500 K. If the Fermi energy in a sample of Si is 0.1 eV and 0.18 eV above the valence band edge at 300 K and 500 K, respectively, calculate the hole concentrations in this sample at these two temperatures.

Solution:

$$N_V (300 \text{ K}) = 2.5 \times 10^{19} \times 0.81^{3/2} \times 1^{3/2} = 1.82 \times 10^{19} \text{ cm}^{-3}.$$

$$N_V (500 \text{ K}) = 2.5 \times 10^{19} \times 0.81^{3/2} \times (500/300)^{3/2} = 3.92 \times 10^{19} \text{ cm}^{-3}.$$

$$p (300 \text{ K}) = 1.82 \times 10^{19} \times \exp\{-0.1 / (8.617 \times 10^{-5} \times 300)\} = 3.8 \times 10^{17} \text{ cm}^{-3}.$$

$$p (500 \text{ K}) = 3.92 \times 10^{19} \times \exp\{-0.18 / (8.617 \times 10^{-5} \times 500)\} = 6.02 \times 10^{17} \text{ cm}^{-3}.$$

Fermi level positions in n-type and p-type semiconductors

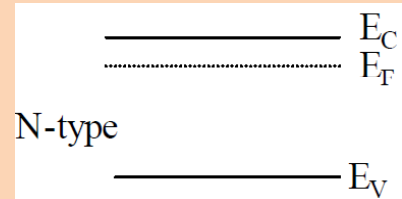


$$n = N_C \exp\left(\frac{E_F - E_C}{KT}\right)$$

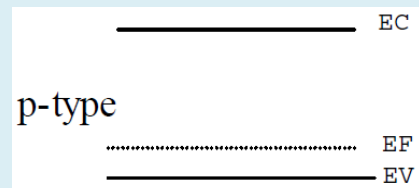
$$p = N_V \exp\left(\frac{E_V - E_F}{KT}\right)$$

N_C and N_V are about the same order of magnitude. Hence, n and p are very sensitive to the Fermi energy E_F (exponential functions).

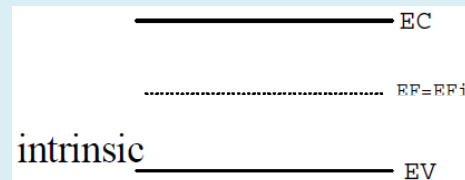
If E_F is closer to E_C (or E_F lies in the upper half of the band gap) then $n \gg p$. The semiconductor is called '**n-type**'.



If E_F is closer to E_V (or E_F lies in the lower half) then $p \gg n$. The semiconductor is called '**p-type**'.



If E_F is **near** the **center** of the band gap, then $n=p$. The semiconductor is then **intrinsic**.



n,p product



Note that the expressions for n and p are valid irrespective of whether the semiconductor is n-type, or p-type or intrinsic. Only E_F position is different for different types of semiconductors.

- Look at the product of n and p . It is independent of the semiconductor type. It depends upon only the basic material properties.

$$np = N_C N_V \exp\left[-\frac{(E_C - E_F + E_F - E_V)}{kT}\right]$$

$$np = N_C N_V \exp\left[-\frac{(E_C - E_V)}{kT}\right]$$

$$np = N_C N_V \exp\left[-\frac{E_G}{kT}\right]$$

Since np product does not depend upon the type of semiconductor it must be same for the intrinsic semiconductor too.

For intrinsic material, $n=p=n_i$,

$$np = n_i^2 = N_C N_V \exp\left\{-\frac{E_G}{kT}\right\}$$

$$\text{or, } n_i = \sqrt{N_C N_V} \exp\left\{-\frac{E_G}{2kT}\right\}$$

Thus $n.p$ product is a constant at a given T and is equal to n_i^2

Intrinsic Fermi level



For The intrinsic semiconductor:

$$n = p = n_i$$

E_F is called E_{Fi} (the intrinsic Fermi level):

$$E_F = E_{Fi}$$

$$n_i = N_C \exp(E_{Fi} - E_C / KT) \quad n_i = N_V \exp(E_V - E_{Fi} / KT)$$

$$N_C \exp(E_{Fi} - E_C / KT) = N_V \exp(E_V - E_{Fi} / KT)$$

$$\frac{\exp(E_{Fi} - E_C / KT)}{\exp(E_V - E_{Fi} / KT)} = \frac{N_V}{N_C}$$

$$\exp\left(\frac{E_{Fi} - E_C - (E_V - E_{Fi})}{KT}\right) = \frac{N_V}{N_C}$$

$$\left(\frac{2E_{Fi} - (E_C + E_V)}{KT}\right) = \ln\left(\frac{N_V}{N_C}\right)$$

$$E_{Fi} - \frac{E_C + E_V}{2} = \frac{KT}{2} \ln\left(\frac{N_V}{N_C}\right)$$

$$E_{Fi} - \frac{E_C + E_V}{2} = \frac{3}{4} KT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

Alternative expressions for n and p



Since:

$$n_i = N_C \exp(E_{Fi} - E_C / KT)$$

$$N_C = n_i \exp(E_C - E_{Fi} / KT)$$

Substitute in

$$n = N_C \exp(E_F - E_C / KT)$$

$$n = n_i \exp(E_C - E_{Fi} / KT) \exp(E_F - E_C / KT)$$

$$n = n_i \exp(E_F - E_{Fi} / KT)$$

Similarly, we can find p

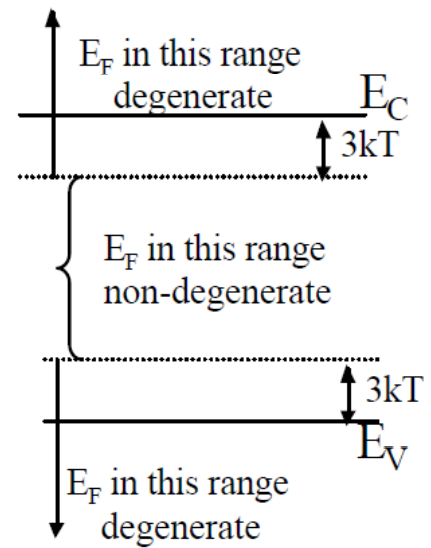
$$p = n_i \exp(E_{Fi} - E_F / KT)$$

It is easier to see that $np = n_i^2$ from the above two equations.

Conditions for the validity of Maxwell - Boltzmann approximation



- M-B approximation is valid only when the Fermi energy is away from either band edge (E_C or E_V) by at least $3kT$.
- $E_C - E_F \gg 3kT$
- or $E_F - E_V \gg 3kT$
- When either of the two conditions is satisfied, then the semiconductor is called a non-degenerate semiconductor.
- A semiconductor in which the neither of the above conditions is satisfied is called a 'degenerate' semiconductor. M-B approximation is not valid for a degenerate semiconductor. The full integral must be evaluated.



Charge neutrality equation and carrier concentrations in doped semiconductors

Charge Neutrality Equation



- So far we have learnt how to calculate the carrier concentrations n and p if we know the Fermi energy E_F relative to E_C or E_V . However, how do we know where is E_F ? In fact, we do not know. What is in our control is the doping concentration. So we need to be able to calculate the carrier concentrations from the impurity concentrations, N_D and N_A .
- The equation underlying the calculation of n or p from N_D and N_A is the charge neutrality equation.

What is charge neutrality equation?

It is simply a statement of the charge neutrality condition. There are **four kinds** of charged species inside a semiconductor; N_D^+ , N_A^- , n^- , p^+ .

A semiconductor in equilibrium is charge neutral everywhere inside the sample. If it were not neutral, then there will be electric fields that will give rise to electric currents against the assumption of equilibrium.

$$n + N_A^- = p + N_D^+$$

Solution of charge neutrality equation (n-type)



Consider the charge neutrality equation:

$$n + N_A^- = p + N_D^+$$

For shallow impurities, $N_D^+ = N_D$ and $N_A^- = N_A$.

N_D and N_A are known from the doping process.

$$n + N_A = p + N_D \quad (1)$$

Still there are two unknowns, n and p . How do we solve for n and p using one equation? Actually there is a second equation that we know between n and p .

$$np = n_i^2 \quad (2)$$

By solving these two equations, we get n and p .

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

Once n is known, p is obtained by using the equation.

$$p = \frac{n_i^2}{n}$$



(n-type) Special cases

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$n = \frac{N_D}{2} + \sqrt{\left(\frac{N_D}{2}\right)^2 + n_i^2} \quad N_A = 0$$

$$n = N_D - N_A \quad \frac{N_D - N_A}{2} \gg n_i \text{ So we can neglect } n_i \text{ w.r.t. } \frac{N_D - N_A}{2}$$

$$n = N_D \quad N_A = 0, N_D \gg n_i$$

Once n is known, p is obtained by using the equation.

$$p = \frac{n_i^2}{n}$$

Solution of charge neutrality equation (p-type)



$$n + N_A = p + N_D \quad (1)$$

Still there are two unknowns, n and p . How do we solve for n and p using one equation? Actually there is a second equation that we know between n and p .

$$np = n_i^2 \quad (2)$$

By solving these two equations, we get p and n .

Eliminating n from equation number (1), so

$$\frac{n_i^2}{p} + N_A = p + N_D$$

$$p^2 + (N_D - N_A)p - n_i^2 = 0$$

Solving: we get
$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

Once n is known, p is obtained by using the equation.

$$n = \frac{n_i^2}{p}$$



(p-type) Special cases

$$p = \frac{N_A - N_D}{2} + \sqrt{\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2}$$

$$p = \frac{N_A}{2} + \sqrt{\left(\frac{N_A}{2}\right)^2 + n_i^2} \quad N_D = 0$$

$$p = N_A - N_D \quad \frac{N_A - N_D}{2} \gg n_i \text{ So we can neglect } n_i \text{ w.r.t. } \frac{N_A - N_D}{2}$$

$$p = N_A \quad N_D = 0, N_A \gg n_i$$

Once **p** is known, **n** is obtained by using the equation.

$$n = \frac{n_i^2}{p}$$

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Solved Examples



Problem:

A silicon sample is doped with $2 \times 10^{15} \text{ cm}^{-3}$ P atoms. If the background acceptor impurity concentration is $1 \times 10^{15} \text{ cm}^{-3}$ calculate the electron and hole concentrations in this sample. Assume complete ionization of all the impurities and the intrinsic carrier concentration in Si at room temperature to be 10^{10} cm^{-3} .

Solution: $n = N_D - N_A = 2 \times 10^{15} - 1 \times 10^{15} = 1 \times 10^{15} \text{ cm}^{-3}$.

$$p = n_i^2/n = (10^{10})^2 / 1 \times 10^{15} = 10^{20} / 10^{15} = 10^5 \text{ cm}^{-3}.$$

Problem:

An n-type Si sample has a donor concentration of 10^{16} cm^{-3} . Suppose we want to convert this sample into p-type with a hole concentration of $5 \times 10^{15} \text{ cm}^{-3}$, what impurity and at what concentration would you use?

Solution: $p = N_A - N_D \Rightarrow N_A = p + N_D = 5 \times 10^{15} + 1 \times 10^{16} = 1.5 \times 10^{16} \text{ cm}^{-3}$. We need to use an acceptor impurity such as B at a concentration of $1.5 \times 10^{16} \text{ cm}^{-3}$.

Fermi level positions relative to intrinsic Fermi level



- Using the expressions for n and p in terms of E_i one can determine the Fermi level position relative to E_i .

- If n is known, then

$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

$$\Rightarrow E_F - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

- If p is known, then

$$p = n_i \exp\left[\frac{(E_i - E_F)}{kT}\right]$$

$$\Rightarrow E_i - E_F = kT \ln\left(\frac{p}{n_i}\right)$$